

The influence of mode mismatch on correlation measurement in a Bell state direct detector

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Received 7 November 2004, accepted for publication 1 March 2005

Published 17 May 2005

Online at stacks.iop.org/JOptB/7/189

Abstract

The influence of mode mismatch on the quantum correlation measurement of phase quadratures of entangled state light beams in a Bell state direct detector is quantitatively discussed. The experimental measurements and theoretical calculation are in good agreement.

Keywords: mode mismatch, quantum correlation, Bell state detector

On the basis of exploitation of the Einstein–Podolsky–Rosen (EPR) entangled state of continuous variables (CV) of the electromagnetic field, a variety of quantum information protocols, such as unconditional quantum teleportation [1], quantum dense coding [2] and entanglement swapping [3], were experimentally demonstrated. Recently, the multipartite entangled states of light were experimentally obtained by combining or splitting squeezed state lights [4–6]. Novel quantum communication networks—for example, controlled dense coding [4], secret quantum state sharing [7] and quantum teleportation networks [8]—were achieved by using tripartite entanglement of light beams. In all quantum information protocols of CV, Bell state detection, that is the joint measurement of amplitude and phase quadratures of two optical modes, is a necessary procedure. Usually, a Bell state measurement instrument consists of a 50% optical beam splitter and two sets of optical homodyne detectors for detecting the joint amplitude quadrature and phase quadrature. To implement balanced homodyne detection, a local oscillator has to be used; thus the complexity of the system and alignment procedure is increased. We designed a relatively simple system for Bell state detection, named a Bell state direct detector [9], with which the correlation variances of the amplitude quadrature sum and phase quadrature difference of two light beams combined on a 50% beam splitter can be simultaneously measured and the local oscillator is not needed. The direct Bell state detector can be used for measuring the anticorrelation of amplitude quadratures and the correlation of phase quadratures between two bright light beams with

nonzero average amplitude. This detection system has been successfully applied in the quantum information protocols of CV and its reliability has been experimentally confirmed [2–4]. Although the alignment procedure for mode matching has been significantly simplified and thus the effect of the mode mismatch has also been minimized because the local oscillator is not required in the Bell state direct detector, the influence of mode mismatch on the correlation measurement of phase quadratures is still extant. According to theoretical quantum optics calculations, the absolute values of the quadrature amplitude and phase correlation of an EPR entangled beam produced from a nondegenerate optical parametric amplifier (NOPA) should be equal [10–13]. However, the measured correlation degree of phase quadratures is always a little smaller than that of the amplitude quadratures in experiments using the direct detector and the measured phase correlation strongly depends on the quality of mode matching between two input optical modes [2–4]. In this paper we will quantitatively discuss the influence of mode mismatch between two detected light beams on the correlation measurements of amplitude and phase quadratures in the Bell state direct detector. The values calculated theoretically on the basis of a simple model of additional vacuum noise are in good agreement with the results of the experimental measurements at different mode match efficiencies.

1. Generation of EPR light beams

The amplitude quadratures (\hat{X}_a, \hat{X}_b) and phase quadratures (\hat{Y}_a, \hat{Y}_b) of two output field modes from a NOPA operating

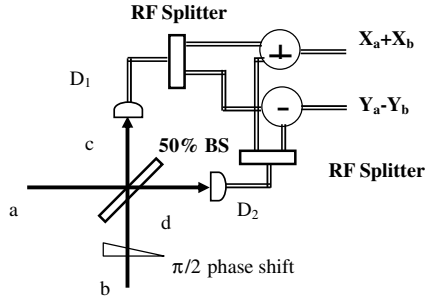


Figure 1. Set-up for the direct measurement of the Bell state system. \hat{a}, \hat{b} : input beam; \hat{c}, \hat{d} : output beam; 50% BS: 50% beam splitter; \oplus : positive power combiner; \ominus : negative power combiner; D_{1-2} : photodiode detector (ETX500 InGaAs).

at deamplification, in which the pump field of the second-harmonic wave and the injected signal field of the fundamental wave are out of phase (π phase difference), are expressed in the frequency domain by [10, 13]

$$\begin{aligned} \hat{X}_a(\Omega) &= \hat{X}_{a0}(\Omega) \cosh r - \hat{X}_{b0}(\Omega) \sinh r \\ \hat{X}_b(\Omega) &= \hat{X}_{b0}(\Omega) \cosh r - \hat{X}_{a0}(\Omega) \sinh r \\ \hat{Y}_a(\Omega) &= \hat{Y}_{a0}(\Omega) \cosh r + \hat{Y}_{b0}(\Omega) \sinh r \\ \hat{Y}_b(\Omega) &= \hat{Y}_{b0}(\Omega) \cosh r + \hat{Y}_{a0}(\Omega) \sinh r \end{aligned} \quad (1)$$

where $\hat{X}_{a0}(\Omega)$, $\hat{X}_{b0}(\Omega)$ and $\hat{Y}_{a0}(\Omega)$, $\hat{Y}_{b0}(\Omega)$ are the amplitude and phase quadratures of two injected signal modes (\hat{a}_0 and \hat{b}_0) with identical frequency and orthogonal polarizations. Modes \hat{a}_0 and \hat{b}_0 are the initial signal and idler modes in a type- Π nonlinear ($\chi^{(2)}$) crystal used for parametric interaction. Ω is the analysis frequency. r ($0 \leq r < \infty$) is the correlation parameter of the output mode \hat{a} (\hat{X}_a, \hat{Y}_a) and \hat{b} (\hat{X}_b, \hat{Y}_b) from the NOPA, which depends on the strength and time of parametric interaction in the NOPA and is a constant parameter under given experiment conditions. From equation (1) we can easily obtain the quantum correlation variances of the amplitude and phase quadratures for two output modes:

$$\begin{aligned} \langle \delta^2(\hat{X}_a + \hat{X}_b) \rangle &= e^{-2r} (\langle \delta^2 \hat{X}_{a0} \rangle + \langle \delta^2 \hat{X}_{b0} \rangle) \\ \langle \delta^2(\hat{Y}_a - \hat{Y}_b) \rangle &= e^{-2r} (\langle \delta^2 \hat{Y}_{a0} \rangle + \langle \delta^2 \hat{Y}_{b0} \rangle). \end{aligned} \quad (2)$$

In experiments, the injected signals are usually coherent states and their fluctuation variances are normalized, i.e. $\langle \delta^2 \hat{X}_{a0} \rangle = \langle \delta^2 \hat{X}_{b0} \rangle = \langle \delta^2 \hat{Y}_{a0} \rangle = \langle \delta^2 \hat{Y}_{b0} \rangle = 1$. For two uncorrelated classical optical modes (two coherent states), we have $r = 0$ and thus $\langle \delta^2(\hat{X}_a + \hat{X}_b) \rangle = \langle \delta^2(\hat{Y}_a - \hat{Y}_b) \rangle = 2$, which is the normalized shot noise limit (SNL) of the correlation variances. When $r > 0$, the correlation variances are smaller than the SNL. In this case, we say that the two optical modes have anticorrelated amplitude quadratures ($\langle \delta^2(\hat{X}_a + \hat{X}_b) \rangle < 2$) and correlated phase quadratures ($\langle \delta^2(\hat{Y}_a - \hat{Y}_b) \rangle < 2$). It is obvious that for a given r the absolute values of $\langle \delta^2(\hat{X}_a + \hat{X}_b) \rangle$ and $\langle \delta^2(\hat{Y}_a - \hat{Y}_b) \rangle$ should be exactly equal.

2. Direct Bell state detector

The schematic diagram of the direct Bell state detector for optical modes is shown in figure 1. The two output light

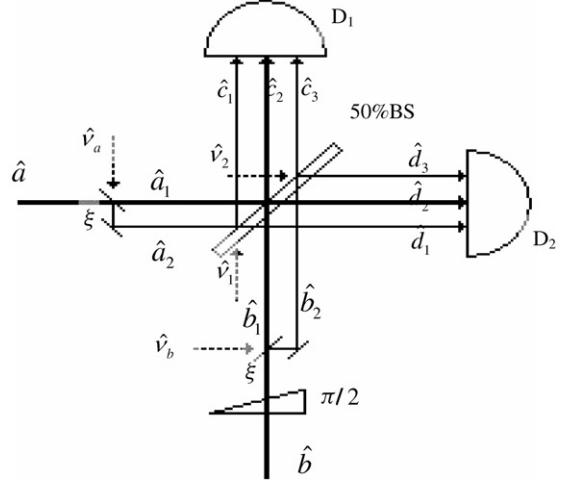


Figure 2. The model for discussing the influence of mode mismatch.

beams from the NOPA, \hat{a} and \hat{b} , are mixed on a 50% beam splitter (BS). Modes \hat{a} and \hat{b} have identical intensity and a phase difference of $\pi/2$ is imposed between \hat{a} and \hat{b} before mixing ($\pi/2$ phase shift). The output modes from the BS, \hat{c} and \hat{d} , are directly detected by photodiodes D_1 and D_2 , respectively. Then, each of the detected photocurrents is divided into two parts through the radio-frequency (RF) power splitter. The variances of the sum (\hat{i}_+) and the difference (\hat{i}_-) of the divided photocurrents at the analysis frequency Ω are expressed by [9]

$$\begin{aligned} \langle \delta^2 i_+(\Omega) \rangle &= \frac{1}{2} \langle \delta^2(\hat{X}_a(\Omega) + \hat{X}_b(\Omega)) \rangle \\ \langle \delta^2 i_-(\Omega) \rangle &= \frac{1}{2} \langle \delta^2(\hat{Y}_a(\Omega) - \hat{Y}_b(\Omega)) \rangle. \end{aligned} \quad (3)$$

In the derivation of equation (3) (see [9]) the effect of mode mismatch was not considered. However, in real systems perfect mode matching is not easy to reach and usually a small mode mismatch is always extant. In quantum optics, the imperfect efficiency of a photodetector ($\eta < 1$) can be modelled as a beam splitter with transmittivity η placed in front of an ideal detector with unity efficiency. This suggests that any loss or inefficiency experienced by a measurement scheme is similar to the interference of the signal with an uncorrelated vacuum field. Therefore, the detrimental effect of mode mismatch on the homodyning detection can be considered to be equivalent to an additional vacuum noise introduced by an optical beam splitter [14]. As shown in figure 2, we can think that an imaginary beam splitter (ξ) is placed in the path of the input light, the transmission efficiency ξ of which is close to 1 but not equal to it; thus the vacuum noises, \hat{v}_a and \hat{v}_b , are introduced in the input modes \hat{a} and \hat{b} respectively. In this case, each of the input modes can be considered as that constituted by two parts \hat{a}_1, \hat{a}_2 and \hat{b}_1, \hat{b}_2 :

$$\begin{aligned} \hat{a}_1 &= \sqrt{\xi} \hat{a} + \sqrt{1-\xi} \hat{v}_a \\ \hat{a}_2 &= \sqrt{1-\xi} \hat{a} - \sqrt{\xi} \hat{v}_a \\ \hat{b}_1 &= \sqrt{\xi} \hat{b} + \sqrt{1-\xi} \hat{v}_b \\ \hat{b}_2 &= \sqrt{1-\xi} \hat{b} - \sqrt{\xi} \hat{v}_b, \end{aligned} \quad (4)$$

where we have taken identical transmission ξ for the two optical modes \hat{a} and \hat{b} . The output modes from the 50% BS should include three parts in each output port [14]:

$$\begin{aligned}
 \hat{c}_1 &= \frac{1}{\sqrt{2}}(\sqrt{1-\xi}\hat{a} - \sqrt{\xi}\hat{v}_a + \hat{v}_1) \\
 \hat{c}_2 &= \frac{1}{\sqrt{2}}(\sqrt{\xi}\hat{a} + \sqrt{1-\xi}\hat{v}_a + i\sqrt{\xi}\hat{b} + i\sqrt{1-\xi}\hat{v}_b) \\
 \hat{c}_3 &= \frac{1}{\sqrt{2}}(\sqrt{1-\xi}\hat{a} - \sqrt{\xi}\hat{v}_a + \hat{v}_2) \\
 \hat{d}_1 &= \frac{1}{\sqrt{2}}(\sqrt{1-\xi}\hat{a} - \sqrt{\xi}\hat{v}_a - \hat{v}_1) \\
 \hat{d}_2 &= \frac{1}{\sqrt{2}}(\sqrt{\xi}\hat{a} + \sqrt{1-\xi}\hat{v}_a - i\sqrt{\xi}\hat{b} - i\sqrt{1-\xi}\hat{v}_b) \\
 \hat{d}_3 &= \frac{1}{\sqrt{2}}(\sqrt{1-\xi}\hat{a} - \sqrt{\xi}\hat{v}_a - \hat{v}_2)
 \end{aligned} \tag{5}$$

where \hat{v}_1 and \hat{v}_2 correspond to the vacuum noises coupled from the other input port of the 50% BS. Calculating the normalized variances of the sum and the difference of the photocurrents from photodiodes D_1 and D_2 , we easily obtain

$$\langle \delta^2 \hat{i}_+(\Omega) \rangle = \frac{1}{2} \langle \delta^2 (\hat{X}_a(\Omega) + \hat{X}_b(\Omega)) \rangle \tag{6a}$$

$$\langle \delta^2 \hat{i}_-(\Omega) \rangle = \frac{1}{2} [\xi^2 \langle \delta^2 (\hat{Y}_a(\Omega) - \hat{Y}_b(\Omega)) \rangle + 2(1 - \xi^2)]. \tag{6b}$$

Equations (6) show the mode mismatch ($\xi < 1$) between \hat{a} and \hat{b} only affects the variance of the phase difference (equation (6b)), while it does not change the variance of the amplitude sum (equation (6a)).

3. Experimental demonstration of the influence of mode mismatch

In the following we will experimentally prove the results of equations (6). The experimental set-up is shown in figure 3. The EPR light beam is produced from a NOPA in a semimonolithic configuration, which consists of an α -cut type- Π KTP crystal and a concave mirror of 50 mm curvature radius. The front face of the KTP crystal was coated for use as the input coupler (the transmission is $>95\%$ for the pump laser at a wavelength of 540 nm and $\sim 0.5\%$ for the injected signal at a wavelength of 1080 nm) and the other face was coated with dual-band antireflection coating at both 540 and 1080 nm. The concave mirror serves as the output coupler of the EPR light beams at 1080 nm generated through the nonlinear interaction (the transmission is 3.8% at 1080 nm and there is high reflection at 540 nm). The output coupler is mounted on a piezoelectric transducer (PZT) for actively locking the cavity length on resonance with the injected signals at 1080 nm using the FM (frequency modulation) sideband technique. The pump laser is a frequency-stabilized and frequency-doubled Nd:YAP laser [15, 16]. The output second-harmonic laser at 540 nm and the sub-harmonic laser at 1080 nm are used as the pump laser and the signal injected into the NOPA, respectively. For obtaining EPR light beams with anticorrelated amplitude quadratures and correlated phase quadratures, the relative phase of the pump laser and the injected signal was locked to $(2n + 1)\pi$ (n is an integer) to enforce the NOPA operation

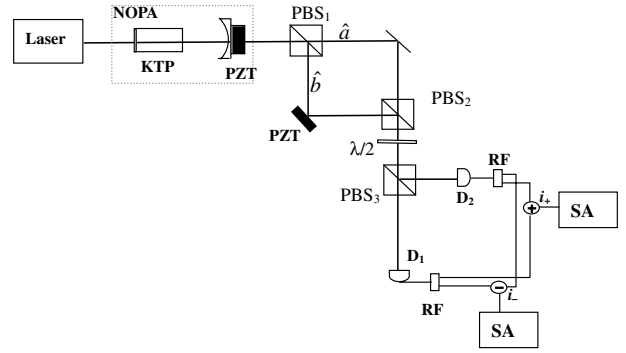


Figure 3. Schematic diagram of the experimental set-up for measuring correlation variances under different mode mismatches. Laser: Nd:YAP/KTP laser; NOPA: nondegenerate optical parametric amplifier; PBS: polarizing beam splitter; SA: spectrum analyser; RF: radio-frequency splitter.

at deamplification conditions. The polarizing beam splitter (PBS_1) separates two EPR beams with identical frequency (1080 nm) and orthogonal polarizations (modes \hat{a} and \hat{b}). A PZT is placed in the path of an optical mode (such as \hat{b}) for aligning the phase difference between mode \hat{a} and \hat{b} to $\pi/2$ to satisfy the requirement of Bell state direct detection. PBS_2 , PBS_3 and a half-waveplate ($\lambda/2$) constitute the 50% beam splitter of the Bell state detector. The photocurrents detected by photodiodes D_1 and D_2 are divided to two parts with identical power by the power splitter (RF), and then the variances of the sum and difference photocurrents are analysed by two spectrum analysers (SA).

The interference efficiency is a physical parameter measuring the coherence of light beams, which equals the ratio of the measured visibility of the interference fringe to the best fringe visibility that can possibly be achieved for two given input light beams [17]. By measuring the interference efficiency of optical modes \hat{a} and \hat{b} on PBS_2 , we can evaluate the quality of the mode match. At first, the mode match was adjusted to the best of our ability, where the interference efficiency of $\sim 99.5\%$ is reached. Then we gradually reduced the interference efficiency and measured the correlation variances of the amplitude sum and phase difference under each condition of mode mismatch. The measured results are listed in table 1. The third and fifth columns give the actual correlation values after accounting for the electronics noise level (ENL) which is 11.3 dB below the SNL. It is obvious that the correlation variances of the amplitude sum ($\langle \delta^2 (\hat{X}_a(\Omega) + \hat{X}_b(\Omega)) \rangle$) almost remained constant when the mode match became worse, but the correlation variances of the phase difference ($\langle \delta^2 (\hat{Y}_a(\Omega) - \hat{Y}_b(\Omega)) \rangle$) monotonically decrease with the reduction of the mode match efficiency. The light reduction of ($\langle \delta^2 (\hat{X}_a(\Omega) + \hat{X}_b(\Omega)) \rangle$) might be occurring because a small part of the light beams is blocked by the frame of the photodiodes for bigger mode mismatch, which corresponds to increasing loss; thus the correlation degrees decrease a little. In our system the diameter of the photodiode frame (ETX500 InGaAs) is about $500 \mu\text{m}$. For small mode mismatch with the match efficiency high than 80% the diameter of the output light spots is smaller than $500 \mu\text{m}$; thus the intensity of the light can be totally detected. But when the mode match efficiency is lower than 80%, some light intensity will be blocked. For

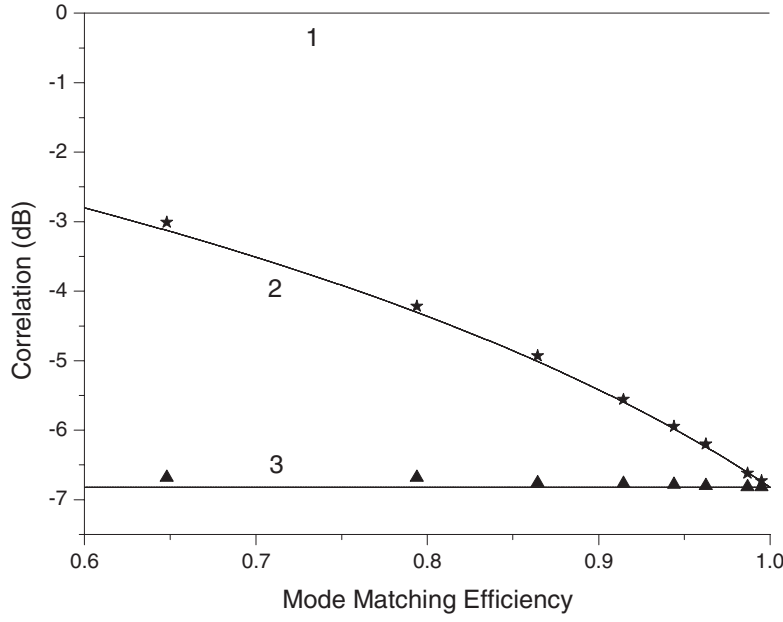


Figure 4. The correlation variances versus mode matching efficiencies. The traces 1, 2, 3 correspond to the SNL and the calculated correlation variances of the phase difference and amplitude sum, respectively. The stars and triangles stand for the measured correlation values of the phase difference and amplitude sum, respectively.

Table 1. The measured correlation variances of the amplitude sum and phase difference.

Mode match efficiency (%)	Correlation variances of amplitude sum ($\delta^2(\hat{X}_a + \hat{X}_b)$)		Correlation variances of phase difference ($\delta^2(\hat{Y}_a - \hat{Y}_b)$)	
	Experimental result (dB)	Result after accounting for ENL (dB)	Experimental result (dB)	Result after accounting for ENL (dB)
99.5	6.05	6.82	5.99	6.73
98.7	6.05	6.82	5.90	6.62
96.1	6.04	6.80	5.56	6.20
94.4	6.03	6.78	5.35	5.95
91.3	6.02	6.77	5.02	5.56
86.0	6.01	6.76	4.50	4.93
79.4	5.99	6.73	3.92	4.27
64.8	5.95	6.68	2.87	3.09

example, when the mode match efficiency equals 70% the measured spot size is about 700 μm which is much larger than the size of the frame.

The comparisons between the theoretical calculations and the experimental measurements of $\langle \delta^2(\hat{X}_a(\Omega) + \hat{X}_b(\Omega)) \rangle$ and $\langle \delta^2(\hat{Y}_a(\Omega) - \hat{Y}_b(\Omega)) \rangle$ for different mode match efficiencies are given in figure 4. The traces 1, 2, 3 correspond to the SNL and the calculated correlation variances of the phase difference and amplitude sum, respectively. The stars and triangles stand for the measured correlation values of the phase difference and amplitude sum, respectively. We see that the theoretical calculation and the experimental measurements are in good agreement.

4. Conclusion

We theoretically analysed and experimentally measured the influence of mode mismatch on the correlation measurements of the amplitude sum and phase difference of EPR beams in a Bell state direct detector. The amplitude correlation is

basically not influenced, but the measured phase correlation decreases with degradation of the mode match efficiency. The results quantitatively explained the experimental phenomenon: that is, the correlation variance phase difference measured with the Bell state detector is always a little smaller than that of the amplitude sum. The discussion provides a useful reference for the processing of experimental results using Bell state detectors.

Acknowledgments

This work was supported by the Major State Basic Research Project of China (No 2001CB309304) and the National Natural Science Foundation of China (Nos 60238010, 60378014).

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